



O'zgaruvchan suv parametrlarida cho'michsimon parrakli mikroGESning optimal ish parametrlarini modellashtirish va eksperimental tadqiq etish

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Dolzarbli: qayta tiklanuvchi energiya manbalaridan, xususan, kichik gidroenergetik salohiyatdan samarali foydalanish energiya talabining ortib borishi sharoitida muhim ahamiyat kasb etmoqda. Past va o'rta naporli suv oqimlarida ishlovchi mikroGESlar mahalliy suv resurslaridan elektr energiyasi olish imkonini beradi. Biroq suv sarfi va naporning o'zgaruvchanligi sharoitida gidroturbinaning optimal ish rejimini aniqlash hamda uning energetik samaradorligini oshirish muhim ilmiy-amaliy masala hisoblanadi. Shu sababli cho'michsimon parrakli mikroGESning suv sarfi, napor, aylanish tezligi va parraklarning geometrik parametrlariga bog'liq optimal ish ko'rsatkichlarini aniqlash dolzarbdir.

Maqsad: tadqiqotning maqsadi o'zgaruvchan suv sarfi va napor sharoitida ishlaydigan cho'michsimon parrakli mikroGES gidroturbinasining optimal ish parametrlarini matematik modellashtirish va eksperimental tadqiqotlar asosida aniqlash, shuningdek, turbina aylanish tezligi, aylantiruvchi moment, mexanik va elektr quvvati hamda foydali ish koeffitsiyenti o'rtaidagi bog'liqliklarni baholashdan iborat.

Usullar: tadqiqotda cho'michsimon parrakli gidroturbina uchun matematik model va hisoblash algoritmi ishlab chiqildi. Suv sarfi, napor, oqim tezligi, parraklar soni, parraklarning o'rnatilish burchagi, aylanish tezligi, aylantiruvchi moment, mexanik quvvat va gidravlik FIK hisoblandi. Optimal ish rejimlarini aniqlashda ko'p bosqichli iteratsion hisoblash va jarima funksiyasi usulidan foydalanildi. Shuningdek, mikroGESning energetik samaradorligini tekshirish uchun laboratoriya va amaliy tajribalar o'tkazilib, suv sarfi, napor, aylanish tezligi, moment hamda mexanik va elektr quvvati o'lchandi. Tajriba natijalari asosida MATLAB muhitida regressiya modeli ishlab chiqildi.

Natijalar: hisoblash va tajriba natijalariga ko'ra, mikroGESning energetik ko'rsatkichlari suv sarfi, napor va turbina aylanish tezligiga sezilarli darajada bog'liq ekanligi aniqlandi. Hisobiy rejimda $H = 30$ m va umumiy $Q = 0,018$ m³/s bo'lganda har bir gidroturbinaning optimal aylanish tezligi 540 rpm, mexanik quvvati 2150 W, ikki turbina uchun umumiy mexanik quvvat esa 4300 W ni tashkil etdi. Tajribalarda $H = 5-30$ m, $Q = 0,0036-0,009$ m³/s va $N = 220-540$ rpm oralig'ida tadqiqot olib borildi. Elektr quvvati napor 5 m da 300 W dan 30 m da 4385 W gacha oshdi. Matematik model va tajriba natijalari o'rtaidagi moslik $R^2 \approx 0,929$ ni tashkil etib, modelning yuqori aniqligini ko'rsatdi.

Kalit so'zlar: mikroGES, cho'michsimon parrakli gidroturbina, qayta tiklanuvchi energiya, suv sarfi, gidravlik napor, aylanish tezligi, aylantiruvchi moment, mexanik quvvat, elektr quvvati, foydali ish koeffitsiyenti, matematik modellashtirish, eksperimental tadqiqot, regressiya modeli, optimal ish rejimi.

Моделирование и экспериментальное исследование оптимальных рабочих параметров микроГЭС с ковшовыми лопатками при переменных параметрах водного потока

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Актуальность: использование возобновляемых источников энергии, в частности малого гидроэнергетического потенциала, приобретает важное значение в условиях постоянно растущего спроса на энергию. МикроГЭС, работающих при низких и средних напорах водных потоков, позволяющих получать электрическую энергию за счёт использования местных водных ресурсов. Однако в условиях изменяющихся расхода воды и напора определение оптимального режима работы гидротурбины и повышение её энергетической эффективности остаются важными научно-практическими задачами. В связи с этим актуальным

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является определение оптимальных рабочих характеристик микроГЭС с ковшовыми лопатками, в зависимости от расхода воды, напора, частоты вращения и геометрических параметров лопаток.

Цель: цель исследования заключается в определении оптимальных рабочих параметров гидротурбины микроГЭС с ковшовыми лопатками, работающей в условиях переменного расхода воды и напора, на основе математического моделирования и экспериментальных исследований, а также в оценке взаимосвязей между частотой вращения турбины, крутящим моментом, механической и электрической мощностью и КПД.

Методы: в исследовании была разработана математическая модель и расчётный алгоритм для гидротурбины с ковшовыми лопатками. Были рассчитаны расход воды, напор, скорость потока, количество лопаток, угол их установки, частота вращения, крутящий момент, механическая мощность и гидравлический КПД. Для определения оптимальных режимов работы применялись итерационные расчёты и метод штрафных функций. Кроме того, для оценки энергетической эффективности микроГЭС были проведены лабораторные и натурные экспериментальные исследования, в ходе которых измерялись расход воды, напор, частота вращения, крутящий момент, а также механическая и электрическая мощность. На основе экспериментальных данных в среде MATLAB была разработана регрессионная модель.

Результаты: по результатам расчётов и экспериментальных исследований установлено, что энергетические показатели микроГЭС в значительной степени зависят от расхода воды, напора и частоты вращения турбины. В расчётном режиме при $H = 30$ м и общем расходе воды $Q = 0,018$ м³/с оптимальная частота вращения каждой гидротурбины составила 540 об/мин, механическая мощность — 2150 Вт, а суммарная механическая мощность двух турбин — 4300 Вт. Эксперименты исследования проводились при $H = 5–30$ м, $Q = 0,0036–0,009$ м³/с и $N = 220–540$ об/мин. Электрическая мощность увеличивалась с 300 Вт, при напоре 5 м, до 4385 Вт при напоре 30 м. Степень соответствия математической модели экспериментальным результатам составила $R^2 \approx 0,929$, что свидетельствует о высокой точности разработанной модели.

Ключевые слова: микроГЭС, гидротурбина с ковшовыми лопатками, возобновляемая энергетика, расход воды, гидравлический напор, частота вращения, крутящий момент, механическая мощность, электрическая мощность, коэффициент полезного действия, математическое моделирование, экспериментальное исследование, регрессионная модель, оптимальный режим работы.

Modeling and Experimental Investigation of the Optimal Operating Parameters of a Bucket-Bladed Micro Hydropower Plant under Variable Water Conditions

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Relevance: the efficient utilization of renewable energy sources, particularly small-scale hydropower potential, is becoming increasingly important amid growing energy demand. Micro-hydropower plants operating under low and medium hydraulic heads provide an opportunity to generate electricity from local water resources. However, under conditions of variable water discharge and hydraulic head, determining the optimal operating regime of the hydraulic turbine and improving its energy efficiency remain important scientific and practical challenges. Therefore, determining the optimal operating performance of a bucket-bladed micro-hydropower plant depending on water discharge, hydraulic head, rotational speed, and blade geometric parameters is highly relevant.

Aim: the aim of this study is to determine the optimal operating parameters of a bucket-bladed micro-hydropower plant turbine operating under variable water discharge and hydraulic head conditions through mathematical modeling and experimental investigations. In addition, the study evaluates the relationships among turbine rotational speed, torque, mechanical and electrical power, and efficiency.

Methods: a mathematical model and computational algorithm were developed for the bucket-bladed hydraulic turbine. Water discharge, hydraulic head, flow velocity, number of blades, blade installation angle, rotational speed, torque, mechanical power, and hydraulic efficiency were calculated. Multi-stage iterative calculations and the penalty function method were used to determine the optimal operating regimes. In addition, laboratory and practical experiments were conducted to evaluate the energy efficiency of the micro-hydropower plant, during which water discharge, hydraulic head, rotational speed, torque, and mechanical and electrical power were measured. Based on the experimental results, a regression model was developed in the MATLAB environment.

Results: the calculation and experimental results showed that the energy performance of the micro-hydropower plant is significantly dependent on water discharge, hydraulic head, and turbine rotational speed. Under the calculated operating conditions of $H = 30$ m and a total flow rate of $Q = 0.018$ м³/с, the optimal rotational speed of each hydraulic turbine was 540 rpm, with a mechanical power output of 2150 W, while the total mechanical power of the two turbines reached 4300 W. The experiments were conducted within the ranges of $H = 5–30$ м, $Q = 0.0036–0.009$ м³/с, and $N = 220–540$ rpm. The electrical power increased from 300 W at a hydraulic head of 5 м to 4385 W at 30 м. The agreement between the mathematical model and experimental results was $R^2 \approx 0.929$,

indicating the high accuracy of the developed model.

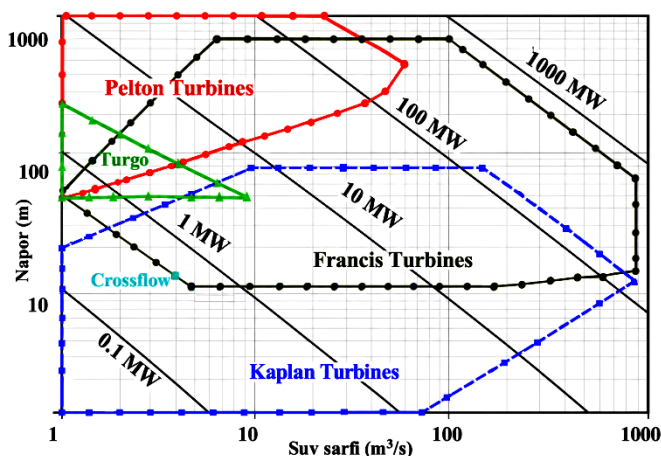
Keywords: micro-hydropower plant, bucket-bladed hydraulic turbine, renewable energy, water discharge, hydraulic head, rotational speed, torque, mechanical power, electrical power, efficiency, mathematical modeling, experimental investigation, regression model, optimal operating regime.

1. Kirish (Introduction)

Energiya talabining ortib borishi va an'anaviy energiya resurslarining cheklanganligi qayta tiklanuvchi energiya manbalaridan, xususan, kichik gidroenergetikadan samarali foydalanishni dolzarb masalaga aylantirmoqda. Past naporli suv oqimlarida ishlovchi mikro GESlar mahalliy gidroenergetik salohiyatdan foydalanish imkonini beruvchi istiqbolli texnologiyalardan biridir [1]. O'zbekistonda irrigatsiya kanallari va kichik suv oqimlarining gidroenergetik salohiyatidan foydalanish mikro GESlarni rivojlantirish uchun katta imkoniyat yaratadi [2]. Biroq past napor va suv sarfining o'zgaruvchanligi sharoitida qurilmaning optimal ish parametrlarini aniqlash hamda energetik samaradorligini oshirish muhim ilmiy-amaliy muammo bo'lib qolmoqda. Mavjud tadqiqotlarda mikro GESlarning konstruksiyasi va turbina parametrlarini optimallashtirish bo'yicha natijalar mavjud bo'lsa-da, past naporli suv oqimlari sharoitida napor, suv sarfi, aylanishlar soni, burovchi moment, quvvat va foydali ish koeffitsiyenti o'rtasidagi bog'liqlikni matematik va eksperimental usullarni birgalikda qo'llagan holda aniqlash masalasi yetarlicha yoritilmagan. Ushbu tadqiqotda past naporli suv oqimlariga moslashtirilgan mikro GES qurilmasining optimal ish parametrlari matematik model-lashtirish va eksperimental tadqiqotlar asosida aniqlanadi. Tadqiqotning asosiy farqi — qurilmaning optimal ish rejimini matematik model natijalarini tajriba ma'lumotlari bilan taqqoslash orqali aso-slashdan iborat.

2. Materiallar va usullar (Methods and materials)

Mikro va kichik GESlarning samaradorligi, avvalo, suv naporli, suv sarfi va oqim tezligiga bog'liq. Shu sababli gidroturbina tanlashda ushbu parametrlar hamda talab etiladigan quvvat asosiy mezon sifatida olinadi (1-rasm). Past naporli mikro va piko GESlarda esa texnik va ekspluatatsion omillarni ham hisobga olish zarur.

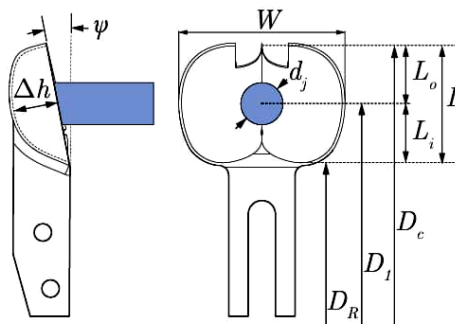


1-rasm. Gidroturbinalar samaradorligining suv naporli va suv sarfiga bog'liqligi
Fig.1. Hydraulic head and water discharge dependence of hydroturbine efficiency

Gidroturbinalar ish sharoitiga ko'ra turlicha qo'llanadi. Banki turbinalari sodda konstruksiyasi, ishlab chiqarish qulayligi va moslashuvchanligi bilan kichik GESlar uchun samarali bo'lib, odatda 2–200 m napor va 0,5–10 m³/s suv sarfi oralig'ida ishlaydi. Frensis turbinalari asosan o'rta va yuqori naporli tizimlarda, Kaplan turbinalari past napor va katta suv sarfi sharoitida, Pelton turbinalari esa yuqori napor va kichik suv sarfi sharoitida qo'llanadi [6-10]. Gidroturbinalar tanlashda suv naporli va sarfi bilan bir qatorda turbina konstruksiyasi hamda ish rejimining mosligi muhim ahamiyatga ega. Yuqori napor va nisbatan kichik suv sarfi sharoitida cho'michsimon parrakli turbinalar istiqbolli yechim bo'lib, keyingi qismda ularning mikro GES uchun asosiy konstruktiv va energetik parametrlari asoslanadi.

Kichik GESlarda energiya samaradorligi suv oqimi energiyasidan foydalanish darajasiga bog'liq. Cho'michsimon parrakli gidroturbinalarda suvning kinetik energiyasi parraklarga uzatilib, mexanik energiyaga aylantiriladi. Turbina samaradorligini belgilovchi asosiy omillar oqim tezligi va yo'nalishi hamda parrak geometriyasidir. Turbina ish jarayonini baholashda matematik va kompyuter model-lashtirish usullaridan foydalaniladi. Matematik model orqali oqim tezligi, zarba burchagi, quvvat, aylantiruvchi moment va FIK aniqlansa, kompyuter model-lashtirish yordamida oqim dinamikasi va

suvning parrak bilan o'zaro ta'siri tahlil qilinadi. Mazkur yondashuvlar turbina parametrlarini optimal lashtirish va uning energetik samaradorligini oshirish imkonini beradi. Tadqiqotda turbina quvvati, suv sarfi, oqim tezligi, parrak geometriyasi va FIK asosiy ko'rsatkichlar sifatida qabul qilindi. Hidroturbina konstruksiyasining asosiy geometrik parametrlari yakka cho'mich misolida 2-rasmda keltirilgan.



2-rasm. Cho'michsimon parrakning geometrik parametrlari

Fig.2. Geometric Parameters of the Pelton Bucket

2-rasmda cho'michsimon parrakning asosiy geometrik parametrlari keltirilgan. Jumladan, D_c — tashqi diametr, m; D_1 — turbina diametri, m; D_R — ichki diametr, m; d_j — soplo chiqish diametri, m; W — parrak eni, m; α — oqimga nisbatan o'rnatilish burchagi, grad; Δh — parrak chuqurligi, m; L — gidravlik yuzaning radial uzunligini ifodalaydi, m (1):

$$L = \frac{D_c - D_R}{2}. \quad (1)$$

Parrakning gidravlik yuzasi bo'yicha uzunliklar L_i va L_o mos ravishda D_1 diametrining ichki va tashqi zonalaridagi qismlarni tavsiflaydi (2), (3).

$$L_i = \frac{D_1 - D_R}{2} \quad (2)$$

$$L_o = \frac{D_c - D_1}{2} \quad (3)$$

Gidroturbina uchun optimal geometrik parametrlar ko'p bosqichli iteratsion hisoblash asosida aniqlanib, bunda Zhang [3] tomonidan taklif etilgan metodikadan foydalanildi. Mazkur metodikaning nazariy asoslari avvalgi tadqiqotlarda batafsil yoritilganligi sababli, ushbu ishda hisoblash natijalari va ularning tahliliga asosiy e'tibor qaratildi.

Cho'michsimon parrakli gidroturbinasida suv energiyasidan samarali foydalanish uchun parrakning aylanish tezligi oqimning kirish tezligining taxminan 50 % ini tashkil etishi optimal hisoblanadi [4]:

$$u = \frac{v_1}{2}. \quad (4)$$

Soplodan chiqishdagi suv tezligi quyidagicha aniqlanadi:

$$v_1 = c_v \sqrt{2gH_0}; \quad (5)$$

bu yerda u — parrakning tangensial tezligi, m/s; v_1 — suv oqimining soplodan chiqish tezligi, m/s; c_v — tezlik koeffitsiyenti; g — erkin tushish tezlanishi, m/s²; H_0 — foydali gidravlik napor, m.

Oqim va parrakning o'zaro harakatini tavsiflovchi nisbiy tezlik quyidagicha ifodalanadi [5,6,7]:

$$u_1 = \omega \frac{D_1}{2} = \omega R_1 = \frac{v_1}{2}; \quad (6)$$

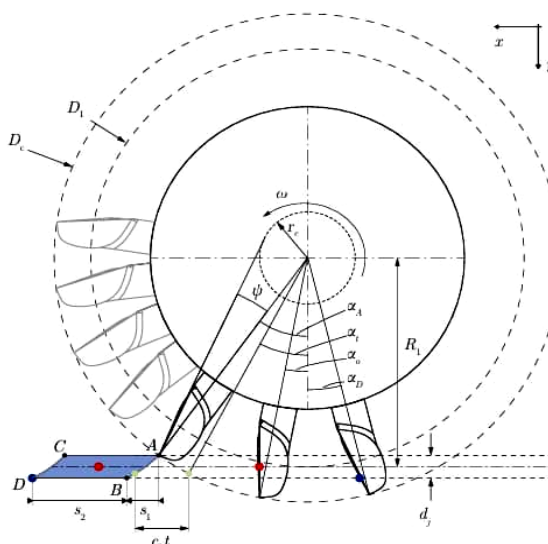
$$w_1 = v_1 - u_1 = \frac{v_1}{2}. \quad (7)$$

Soplo chiqishidagi suv oqimi diametri esa quyidagi ifoda orqali aniqlanadi [8,9]:

$$d_j = \frac{Q}{Z \sqrt{2gH_e}}; \quad (8)$$

bu yerda ω — burchak tezlik, rad/s; d_j — soplo chiqish diametri, m; Q — suv sarfi, m³/s; Z — soplolar soni.

Cho'michsimon parrakli turbina gidrodinamik jarayonlarini tahlil qilishda x-y koordinatalar tizimi qo'llanildi. Ushbu yondashuv yordamida suv oqimi va parrakning o'zaro ta'siri natijasida hosil bo'luvchi tangensial va radial tezlik komponentlari, shuningdek, oqimning impuls almashinuvi va aylantiruvchi momenti baholandi. 3-rasmda soplodan chiqayotgan suv oqimining cho'michsimon parrak bilan o'zaro ta'sirining geometrik sxemasi keltirilgan. Oqim parrakning boshlang'ich qismiga α_1 burchak ostida kirib, parrak yuzasi bo'ylab harakatlanadi. Ushbu jarayon turbina ishchi g'ildiragiga uzatiladigan aylantiruvchi moment va gidravlik energiyani belgilovchi asosiy omillardan biridir.



3-rasm. Cho'michsimon parrak va soplodan chiqayotgan suv oqimining o'zaro ta'sir geometrik sxemasi.

Fig.3. Geometric Schematic of the Interaction between the Pelton Bucket and the Water Jet Discharged from the Nozzle

Parrakning A nuqtadagi holati asosida soplodan chiqayotgan suv oqimining geometrik parametrlari quyidagi tenglamalar orqali aniqlanadi:

$$\cos(\alpha_A) = \frac{R_1 - \frac{d_j}{2}}{R_c} \quad (9)$$

$$S_1 = \frac{d_j \left(\frac{1}{u_1} - 1 \right)}{\sqrt{\left(\frac{R_c}{R_1} \right)^2 - 1}} \quad (10)$$

$$S_2 = \frac{R_1}{u_1} \frac{2\pi}{N} \quad (11)$$

bu yerda α_A — suv oqimi va parrakning dastlabki tutashuv burchagi, grad; R_1 — turbina radiusi, m; R_c — turbina tashqi radiusi, m; S_1 — oqimning parrak bilan dastlabki tutashuvigacha bo'lgan uzunligi, m; S_2 — oqimning keyingi parrak bilan tutashuvigacha bo'lgan masofa, m; N — cho'michsimon parraklar soni.

(12)-tenglama suv zarrachasining soplodan chiqib, oqim bo'ylab harakatlanishi bilan turbina parragining burchak holati o'rtasidagi bog'lanishni ifodalaydi. U parrak va suv oqimining o'zaro ta'sir holatini aniqlashda qo'llaniladi. A nuqta koordinatalar boshi sifatida qabul qilinib, x va y koordinatalari suv zarrachasining joriy holatini belgilaydi:

$$\frac{x}{R_1} = \frac{\alpha_A - \alpha_t}{u_1} - \left(1 - \frac{d_j}{2R_1} \right) \tan(\alpha_A) + \left[1 - \frac{d_j}{2R_1} + \frac{y}{R_1} + \frac{r_e}{R_1 \sin(\alpha_t - \psi)} \right] \tan(\alpha_t - \psi); \quad (12)$$

bu yerda x/R_1 va y/R_1 — oqim nuqtasining nisbiy koordinatalari; α_t — keyingi ta'sir nuqtasining burchak koordinatasi, grad; α_A — boshlang'ich oqim burchagi, grad; ψ — parrakning og'ish burchagi, grad; r_e — turbina markazi bilan parrak uchi orasidagi gorizontali masofa, m.

Geometrik bog'lanish quyidagicha aniqlanadi:

$$r_e = R_c \tan \psi. \quad (13)$$

α_A kichik bo'lgan holatda α_t ni aniqlash uchun (12) – (14)-tenglamalardan foydalanish mumkin:

$$\alpha_t = \frac{\frac{x}{R_1} - \frac{\alpha_A}{u_1} - \frac{R_c \sin \psi}{R_1} + \left(1 - \frac{d_j}{2R_1} \right) \tan(\alpha_A) + \left(1 - \frac{d_j}{2R_1} + \frac{y}{R_1} \right) \psi}{1 - \frac{1}{u_1} - \frac{d_j}{2R_1} + \frac{y}{R_1}}. \quad (14)$$

Cho'michsimon parrakning suv oqimi bilan o'zaro ta'sirlashish holatidagi α_t burchak (15)-tenglama asosida aniqlanadi. Bunda geometrik koordinatalar quyidagicha qabul qilinadi:

$$x = \frac{S_1 + S_2}{2}, \quad \text{va} \quad y = \frac{d_j}{2};$$



$$\alpha_0 = \frac{\frac{S_1 + S_2}{2R_1} - \frac{\alpha_A}{u_1} - \frac{R_c \sin \psi}{R_1} + \left(1 - \frac{d_j}{2R_1}\right) \tan \alpha_A + \psi}{\left(1 - \frac{1}{u_1}\right)}. \quad (15)$$

Turbina kirishidagi suv oqimining gidravlik quvvati suv sarfi va foydali napor orqali aniqlanadi [10]:

$$P_j = \rho g Q H; \quad (16)$$

bu yerda P_j — suv oqimining gidravlik quvvati, kW; ρ — suv zichligi, kg/m³; Q — suv sarfi, m³/s; H — foydali gidravlik napor, m.

Turbina mexanik quvvati cho'michsimon parrakning kirish va chiqishidagi tangensial tezlik komponentlari farqi asosida aniqlanadi [11,12]:

$$P_t = \rho Q u (V_1 \cos \alpha_0 - u)(1 + \cos \beta). \quad (17)$$

Gidroturbina validagi moment, burchak tezlik va massaviy sarf quyidagicha ifodalanadi [13,14]:

$$P_t = M \omega = \dot{m} u (V_{u1} - V_{u2}); \quad (18)$$

$$M = \frac{P_t}{\omega}; \quad \omega = \frac{u}{r}; \quad (19)$$

$$\dot{m} = \rho Q. \quad (20)$$

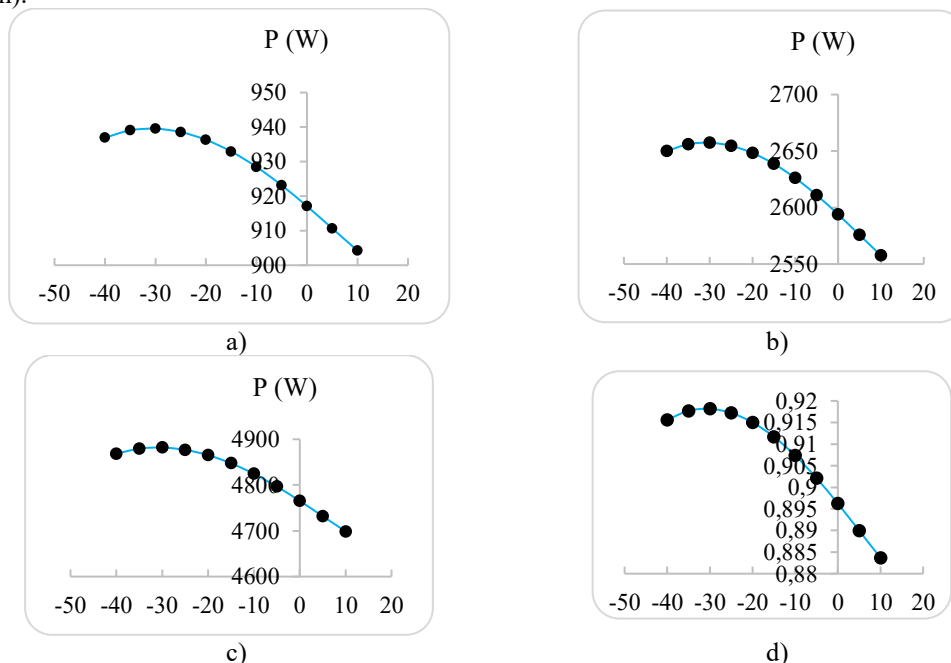
Shundan kelib chiqib, cho'michsimon parrakli mikroGESning gidravlik samaradorligi quyidagicha aniqlanadi:

$$\eta = \frac{P_t}{P_j} = \frac{u(V_1 \cos \alpha_0 - u)(1 + \cos \beta)}{gH}. \quad (21)$$

bu yerda P_t — mexanik quvvat, kW; M — aylantiruvchi moment, N·m; ω — burchak tezlik, rad/s; $\dot{m}$ — massaviy sarf, kg/s; V_{u1} va V_{u2} — kirish va chiqishdagi tangensial tezlik komponentlari; η — gidravlik samaradorlik; β — oqimning parrakdan chiqish burchagi, grad.

3. Natijalar (Results)

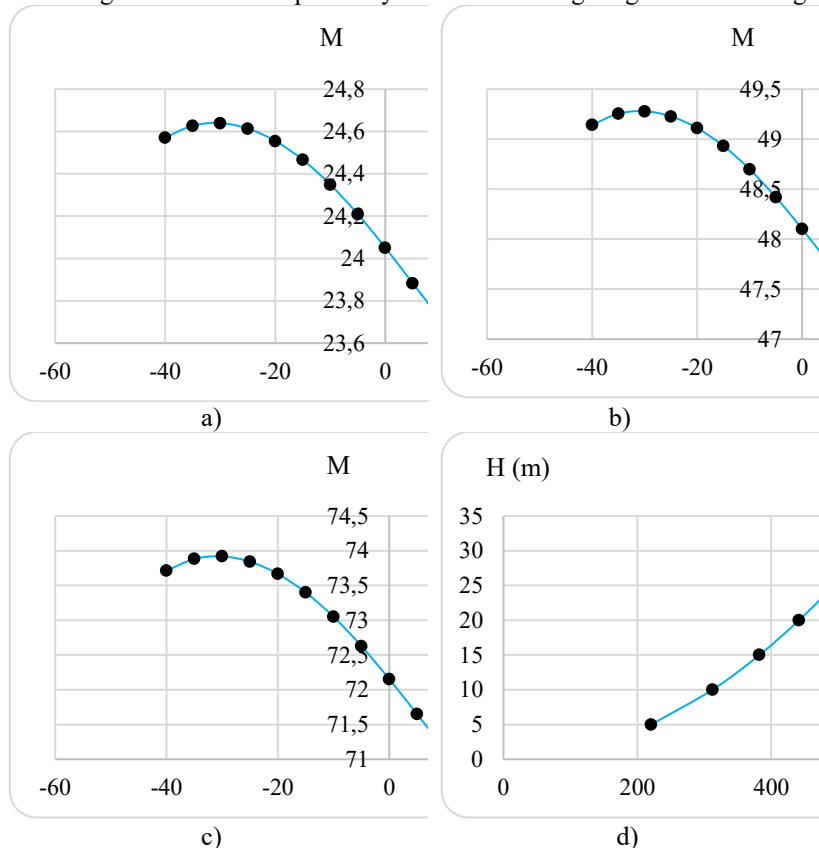
Yuqorida keltirilgan matematik model asosida cho'michsimon parrakli gidroturbinasida suv oqimi kinetik energiyasining mexanik energiyaga aylanishi, aylanish tezligi va inersiya ta'siri nazariy jihatdan baholandi. Model mikro va kichik GESlar uchun turbina parametrlarini, jumladan, suv sarfi, napor, parraklar soni va parrakning o'rnatilish burchagini optimallashtirish imkonini beradi. 4-rasmda turli napor va suv sarfi sharoitlarida parrakning o'rnatilish burchagining mexanik quvvat va foydali ish koeffitsiyentiga (FIK) ta'siri keltirilgan. Hisoblashlar $H = 10\text{--}30$ m va $Q = 0,005\text{--}0,009$ m³/s oralig'idagi boshlang'ich shartlar asosida amalga oshirildi. Natijalarga ko'ra, $H = 10$ m, $Q = 0,005$ m³/s; $H = 20$ m, $Q = 0,007$ m³/s va $H = 30$ m, $Q = 0,009$ m³/s holatlarida mexanik quvvat parrakning ma'lum optimal o'rnatilish burchagida maksimal qiymatga erishdi (4a–c-rasmlar). Shuningdek, parrakning optimal o'rnatilish burchagida turbina FIKning yuqori qiymatga yaqinlashishi aniqlandi (4d-rasm).



4-rasm. Cho'michsimon parraklarning o'rnatish burchagiga nisbatan turbina quvvati va FIKning o'zgarishi.

Fig.4. Effect of the Bucket Installation Angle on Turbine Power and Efficiency

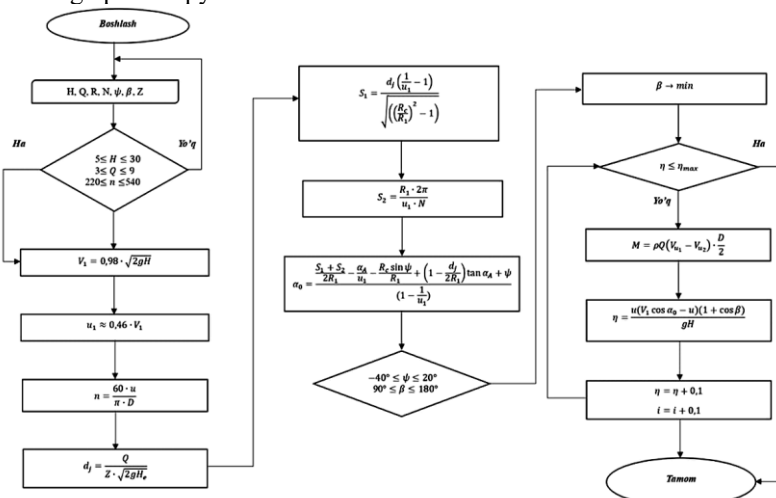
5-rasmda suv oqimi naporining gidroturbina momenti va parraklarning aylanishlar soniga ta'siri keltirilgan. Hisoblashlar turli napor va suv sarfi qiymatlaridagi boshlang'ich shartlar asosida amalga oshirildi. Natijalarga ko'ra, 5a–c-rasmlarda mos ravishda turli napor va suv sarfi sharoitlarida turbina validagi maksimal aylantiruvchi moment qiymatlari aniqlangan. 5d-rasmda esa naporning oshishi bilan maksimal momentga mos keluvchi optimal aylanishlar sonining o'zgarishi ko'rsatilgan.



5-rasm. Cho'michsimon parraklarning o'rnatish burchagiga bog'liq turbina momenti hamda naporning parraklar aylanish tezligiga ta'siri

Fig.5. Effects of Bucket Installation Angle on Turbine Torque and Hydraulic Head on Bucket Rotational Speed

Modellashtirish natijalari asosida suv sarfi va turbina aylanish chastotasining napor, aylantiruvchi moment, mexanik quvvat, foydali ish koeffitsiyenti (FIK) hamda parraklarning optimal o'rnatilish burchagiga ta'sirini baholovchi optimallashtirish algoritmi ishlab chiqildi (6-rasm). Maqbul ish rejimini aniqlashda jarima funksiyasi (penalty function) usuli qo'llanilib, berilgan cheklovlar ostida turbina parametrlarining optimal qiymatlari tanlandi.



6-rasm. Suv oqimi sarfi va turbina aylanma tezligining $(H, Q, n, M, P, \eta, \psi_{opt})$ parametrlarga bog'liqlik algoritmi

Fig.6. Algorithm for Determining Turbine Rotational Speed and Water Flow Rate as Functions of H, Q, n, M, P, η and ψ_{opt}

O'tkazilgan hisoblashlar asosida cho'michsimon parrakli gidroturbinaning napor, aylantiruvchi moment, mexanik quvvat, FIK va parraklarning optimal o'rnatilish burchagining suv sarfi hamda turbina aylanish tezligiga bog'liqligi aniqlandi. Tadqiqotda gidroturbina diametri 0,42 m, suv g'ildiragi diski diametri 0,36 m va parraklar soni 25 ta qilib qabul qilindi. Suv sarfining 0,003–0,009 m³/s oralig'idagi qiymatlari ishlab chiqilgan matematik model asosida hisoblandi. Hisoblash natijalariga ko'ra, $Q = 0,009 \text{ m}^3/\text{s}$ bo'lganda parrakning geometrik jihatdan optimal o'rnatilish burchagi $\psi_{opt} = -30^\circ$ ekanligi aniqlandi.

1-jadval taklif etilayotgan cho'michsimon parrakli mikroGESning turli suv sarfi va napor qiymatlariga mos optimal ish parametrlari

Table 1. Optimal Operating Parameters of the Proposed Pelton-Bucket Micro-Hydropower Plant under Various Water Discharge and Hydraulic Head Conditions

N _o	Q ₁ , m ³ /s	n ₁ , rpm	P ₁ , W	H, m	$\psi_{opt,grad.}$	Q ₂ , m ³ /s	n ₂ , rpm	P ₂ , W
1	0,009	540	2150	30	-30	0,009	540	2150

O'tkazilgan hisoblashlar natijasida $Q = 0,018 \text{ m}^3/\text{s}$ va $H = 30 \text{ m}$ sharoitida har bir gidroturbina uchun optimal aylanish tezligi 540 rpm, mexanik quvvat esa 2150 W ekanligi aniqlandi. Ikki turbina uchun generatorga uzatiladigan umumiy mexanik quvvat 4300 W ni tashkil etdi. Vtulkali uzatma orqali turbinalarining qarama-qarshi aylanishi natijasida generatorning aylanish chastotasi 1080 rpm gacha oshirildi.

Ushbu natijalar asosida o'zgaruvchan suv sarfi va past napor sharoitlarida mikroGESning samarali ishlashini ta'minlovchi aylanish tezligi, napor, aylantiruvchi moment, mexanik quvvat, FIK va parraklarning optimal o'rnatilish burchagi aniqlandi.

Qurilmaning energetik samaradorligini amaliy sharoitda baholash maqsadida tajriba tadqiqotlari o'tkazildi (7-rasm). Tajriba qurilmasi suv ta'minoti tizimi, rostlanuvchi soplo, cho'michsimon parrakli ishchi g'ildirak, mexanik uzatma, generator, elektr yuklama va o'lchov vositalaridan tashkil topdi. Tajribalar davomida suv sarfi, napor, turbina aylanish tezligi, aylantiruvchi moment, mexanik quvvat va elektr quvvati o'lchanib, mikroGESning optimal ish rejimlari hamda umumiy samaradorligi baholandi.



7-rasm. Mikro gidroenergetik qurilmasini yig'ish va tajriba-sinov ishlari jarayoni

Fig.7. Assembly and Experimental Testing Process of the Micro-Hydropower System

2-jadvalda mikroGESning amaliy sharoitdagi sinov natijalari keltirilgan. Tajribalar $H = 5\text{--}30\text{ m}$ napor oralig'ida o'tkazilib, magnitoelektrik generatorning energetik ko'rsatkichlari baholandi. Natijalarga ko'ra, naporning ortishi bilan elektr quvvati muntazam oshib, 5 m naporda 300 W, 30 m naporda esa 4300 W ga yetdi. Ushbu o'zgarish turbina aylanish tezligi va aylantiruvchi momentning ortishi bilan izohlanadi.

Tajriba natijalari taklif etilayotgan mikroGESning o'zgaruvchan gidravlik sharoitlarda barqaror ishlashi va past hamda o'rta naporli suv manbalaridan samarali foydalanish imkoniyatini tasdiqladi.

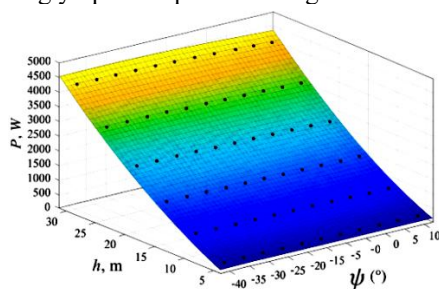
2-jadval. Taklif etilayotgan cho'michsimon parrakli mikroGESning suv napori va suv sarfiga bog'liq optimal ish parametrlari

Table 2. Optimal Operating Parameters of the Proposed Pelton-Bucket Micro-Hydropower Plant as a Function of Hydraulic Head and Water Discharge

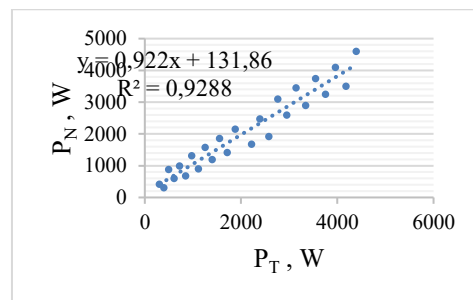
N_2	$H\text{ (m)}$	ψ	$Q\text{ (m}^3\text{/s)}$	$N\text{ (rpm)}$	$M\text{ (Nm)}$	$P\text{ (W)}$
1.	5	-30°	0,0036	220	11,8	300
2.	6		0,004	241	14,3	395
3.	7		0,0043	261	16,8	490
4.	8		0,0046	279	19,4	610
5.	9		0,0049	296	21,9	730
6.	10		0,0052	312	24,4	845
7.	11		0,0054	327	27	970
8.	12		0,0057	341	29,5	1100
9.	13		0,0059	355	32	1255
10.	14		0,0061	369	34,5	1410
11.	15		0,0063	382	36,8	1550
12.	16		0,0065	394	37,9	1700
13.	17		0,0068	406	40,6	1880
14.	18		0,0069	418	43,3	2050
15.	19		0,0071	430	46	2200
16.	20		0,0073	441	48,7	2405
17.	21		0,0075	452	51,3	2570
18.	22		0,0077	462	54	2760
19.	23		0,0079	473	56,5	2955
20.	24		0,008	483	59	3140
21.	25		0,0082	493	61,5	3345
22.	26		0,0084	503	63,9	3540
23.	27		0,0085	512	64	3760
24.	28		0,0087	522	66,8	3965
25.	29		0,0088	531	69,7	4185
26.	30		0,009	540	72,6	4385

Qurilmaning elektromagnit quvvati P ning suv napori H va parraklarning o'rnatilish burchagiga α bog'liqligi bo'yicha tajriba natijalari umumlashtirilib, MATLAB muhitida regressiya tenglamasi ishlab chiqildi. 8-rasmda regressiya modeli asosida $P(\alpha, H)$ funksiyasining fazoviy grafigi (a) hamda tajriba natijalari bilan matematik modelning taqqoslanishi (b) keltirilgan.

Modelning tajriba natijalari bilan moslik darajasi $R^2 = 0,92$ ni tashkil etib, regressiya tenglamasi-ning yuqori aniqlikda ekanligini ko'rsatdi.



a)



b)

8-rasm. Regressiya tenglamasi asosida $f(x, y)$ funksiyaning fazoviy grafigi (a), tajriba natijalari va matematik modelning solishtirilishi grafigi (b)

Fig.8. Spatial Representation of the Function $f(x, y)$ Derived from the Regression Equation (a) and Comparison of Experimental Data with the Mathematical Model (b)



MikroGESning chiqish quvvati va samaradorligining suv nabori (H) hamda parraklarning o'rnatilish burchagi (ψ) ga bog'liqligini aniqlash maqsadida tajriba natijalari asosida ikki o'zgaruvchili regressiya modeli ishlab chiqildi. Tajriba natijalarini matematik ifodalash uchun uchinchi darajali Poly33 polinomial model tanlandi.

Natijada chiqish quvvatini ifodalovchi regressiya tenglamasi quyidagi ko'rinishga ega:

$$(\psi, H) = -98 - 0,13\psi + 51H - 0,017\psi^2 - 0,28\psi H + 3,31H^2 - 0,0007\psi^3 - 0,059\psi^2 H - 0,004H^2\psi - 0,028H^3. \quad (22)$$

Cho'michsimon parrakli mikroGESning fermer xo'jaligi sharoitida joriy etilishining iqtisodiy samaradorligi diskont usuli asosida baholandi (3-jadval). Hisoblash natijalariga ko'ra, NPV 615 mln so'm, IRR 142 %, PI 15,8, LCOE 15 so'm/kWh ni tashkil etdi. Statik qoplash muddati 1 yildan kam, dinamik qoplash muddati esa 1,2 yil deb aniqlandi.

3-jadval. Iqtisodiy samaradorlik ko'rsatkichlari

Table 3. Economic Efficiency Indicators

1.	Sof joriy qiymat (NPV), mln. so'm	615
2.	Foydaning ichki me'yori (IRR), %	142
3.	Foyda indeksi (PI)	15,8
4.	Elektr energiyaning me'yorlashtirilgan (keltirilgan) qiymati (LCOE), so'm/kWh	15
5.	Statik qoplash muddati, yil	< 1
6.	Dinamik qoplash muddati, yil	1,2

Shuningdek, 4,3 kW quvvatli mikroGESni joriy etish natijasida yiliga o'rtacha 2,54 ming AQSh dollari iqtisodiy tejamkorlikka erishish hamda 4,63 tonnadan ortiq CO₂ emissiyasining oldini olish mumkinligi aniqlandi.

4. Munozara (Discussion)

Olingan natijalar cho'michsimon parrakli mikroGESning energetik samaradorligi suv sarfi, napor, turbina aylanish tezligi va parraklarning o'rnatilish burchagiga bevosita bog'liqligini ko'rsatdi. Hisoblash natijalarida napor va suv sarfining ortishi bilan aylantiruvchi moment hamda mexanik quvvatning oshishi aniqlandi. Shuningdek, parraklarning optimal o'rnatilish burchagini tanlash turbina ish samaradorligini oshirish imkonini berdi. Tajriba natijalarida napor $H = 5-30$ m oralig'ida oshirilganda elektr quvvati 300 W dan 4300 W gacha ortdi. Bu natijalar matematik modellashirishda aniqlangan qonuniyatlarni tasdiqladi. Tajriba ma'lumotlari asosida ishlab chiqilgan regressiya modelining $R^2 = 0,92$ qiymati modelning yetarli darajada aniqligini ko'rsatdi. Iqtisodiy baholash natijalariga ko'ra, mikroGESning $NPV = 615$ mln so'm, $IRR = 142\%$, $PI = 15,8$ va dinamik qoplash muddati 1,2 yilni tashkil etdi. Ushbu ko'rsatkichlar taklif etilayotgan mikroGESni amaliy sharoitlarda qo'llash iqtisodiy jihatdan maqsadga muvofiqligini ko'rsatadi.

5. Xulosa (Conclusion)

1. O'zgaruvchan suv sarfi va napor sharoitida ishlaydigan cho'michsimon parrakli mikro GES gidroturbinasi uchun matematik model va hisoblash algoritmi ishlab chiqildi. Modelda suv sarfi 0,0036–0,009 m³/s, napor 5–30 m, turbina aylanish tezligi 220–540 rpm oralig'ida tadqiq qilinib, gidroturbinaning energetik ko'rsatkichlari va optimal ish rejimlari aniqlandi.

2. Hisoblash natijalariga ko'ra, gidroturbina konstruksiyasida parraklarning geometrik parametrlari va optimal o'rnatilish burchagi suv oqimidan energiya olish samaradorligiga sezilarli ta'sir ko'rsatishi aniqlandi. Ishchi g'ildirak diametri 0,42 m, disk diametri 0,36 m va 25 ta cho'michsimon parrakdan iborat konstruksiya uchun suv sarfi va naporning turli qiymatlarida maqbul ish rejimi belgilandi.

3. Napor 30 m va umumiy suv sarfi 0,018 m³/s bo'lgan hisobiy rejimda har bir gidroturbinaning aylanish tezligi 540 rpm, mexanik quvvati 2150 W ni tashkil etdi. Ikki gidroturbinadan generatorga uzatiladigan umumiy hisobiy mexanik quvvat 4300 W ga yetdi. Qarama-qarshi yo'nalishda aylanuvchi gidroturbinalarda vtulkali uzatmadan foydalanish generator aylanish chastotasini 1080 rpm gacha oshirish imkonini berdi.

4. Laboratoriya va amaliy tajribalarda napor 5–30 m, suv sarfi 0,0036–0,009 m³/s va turbina aylanish tezligi 220–540 rpm oralig'ida tekshirildi. Tajriba natijalariga ko'ra, generator quvvati napor

5 m da 300 W dan napor 30 m da 4385 W gacha oshdi. 30 m napor, 0,009 m³/s suv sarfi, 540 rpm aylanish tezligi rejimida aylantiruvchi moment 72,6 N·m ni tashkil etdi.

5. Matematik model natijalarining tajriba ma'lumotlari bilan mosligi yuqori bo'lib, ap-proksimatsiya ishonchliligi $R^2 \approx 0,929$ ni tashkil etdi. Bu ishlab chiqilgan matematik modelning gidro-turbina energetik parametrlarini baholash va optimal ish rejimlarini tanlashda amaliy qo'llash mum-kinligini ko'rsatdi.

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